SOUTH AFRICAN RESERVE BANK

Prudential Authority

Discussion paper on Prudential Authority Government Bond Curve Review

Prudential Authority Government Bond Curve Review

Executive summary

The Insurance Act 18 of 2017 (Insurance Act) mandates the Prudential Authority (PA) to publish a government bond curve, as prescribed in Prudential Standard FSI 2.2, which insurers must use as the risk-free interest rate term structure to discount cash-flows for the purposes of valuing technical provisions.

In this discussion document, the current methodology and data set, which underlie the published PA government bond curve, are presented along with a review of both the methodology and data set.

The first part of the review presents the data set management framework, and further enhancements to this framework are proposed to arrive at an optimal data set based on guiding principles.

The second part presents alternative curve construction methodologies and their relative merits in comparison to the methodology used by the PA (Linear interpolation and extrapolation). In particular, the Cubic spline, Monotone convex, Nelson-Siegel, Nelson-Siegel Svensson and Smith-Wilson methodologies are introduced.

The recommendation includes a constituent data set management proposal as well as revision to the nominal ultimate forward rate (UFR) to be considered. In addition to this, it is recommended that the alternative curve construction methodologies should be considered. Should the alternative methods not be supported by industry participants the current PA methodology should be retained given its relative merits of simplicity.

1. Problem statement and aim of the review

1.1 Introduction

1.1.1 This document aims to set out the basis for, and the findings of, a review conducted about the current curve construction methodology (methodology) and constituent data set (data set) that underlie the PA government bond curve (PA curve) which is published monthly. Alternative curve construction methodologies have been researched with the aim to compare with the current methodology and recommend changes as deemed necessary to achieve an optimal methodology and data set to adopt.

1.1.2 In terms of section 63(1)(a) of the Insurance Act, the PA may prescribe Prudential Standards on any matter that is required or permitted to be prescribed in terms of the Insurance Act.

1.1.3 Paragraphs 13.1 and 13.2 of Prudential Standard FSI 2.2 [Valuation of Technical Provisions] state that:

"13.1. Unless otherwise approved by the Prudential Authority, insurers must use the government bond curve published by the Prudential Authority as the risk-free interest rate term structure to discount cash-flows for the purposes of valuing technical provisions.
13.2. An insurer may apply to the Prudential Authority to use an alternative interest rate term structure to discount cash-flows on liabilities that are matched with swap-based assets, and where those liabilities are valued using the relevant swap curve. If the Prudential Authority grants approval to use a swap curve, the swap curve must be constructed by the insurer using observable market data and must not include any margins for credit or liquidity risk."

1.1.4 The PA methodology was developed in accordance with the principles of the Solvency Assessment and Management (SAM) framework. The principles, in turn, are aligned with the Insurance Core Principles (ICPs) published by the International Association of Insurance Supervisors (IAIS).

1.1.5 Starting in 2012 and following the first quantitative impact study (QIS1) by the then Financial Services Board (FSB), a government bond curve has been published on the first working day of each month. Since then, the then FSB, and subsequently the PA, has monitored the published government bond curve and engaged, from time to time, with various market participants who use the published curves. The main observations and industry feedback are detailed in section 1.4 below.

1.2 Problem statement

1.2.1 In line with the principles of good governance, which by implication suggests that a regular review of regulatory instruments is desirable, a review of the methodology, including the data set, was undertaken by the PA.

1.2.2   Movements in the published PA curve are monitored, from month to month, in relation to the market forces affecting the yields on the instruments in the data set. For example, some volatility was observed in the PA curve caused by market responses to both the coronavirus 2019 (COVID-19) pandemic and a sovereign ratings downgrade in 2020. The sensitivity of the PA curve to these changes was exacerbated by a combination of the construction methodology and data set. Annexure A shows the history of the data set used in the published government bond curve since December 2012.

1.3   Aim of the review

1.3.1   In January 2018, the SAM Policy Steer Committee published the policy steer document on the financial soundness standards, which incorporated the principles by which the various approaches available in the South African context were weighed up in deciding on an appropriate approach. These principles are quoted in section 3.1 below. The published PA curve satisfies these guiding principles. The review is therefore limited to the construction methodology and data set of the government bond curve.

1.3.2   The current construction methodology of the PA curve has not been previously documented, and this has been addressed as part of this review.

1.3.3   To satisfy these aims, the review has been split into two parts:

1.3.3.1   A review of the data set, which covers the inclusion criteria for instruments and their entry and exit criteria.
1.3.3.2   A review of different curve construction methodologies, including interpolation, bootstrapping and extrapolation techniques as well as choices for the UFR.

1.4   Key observations from industry feedback

1.4.1   The following observations have been made based on feedback from the industry using the PA curve:

1.4.1.1   As the current methodology was previously not documented, it   presents a challenge for insurers who seek to replicate the PA curve and produce it at intervals more frequent than monthly. This is addressed as outlined in paragraph 1.3.2 above.
1.4.1.2   In 2020, the level of the PA curve was generally above the spot yields observed in the market, even when compared to forward rates. This observation is more pronounced during periods of market volatility where the movements in the PA curve tend to be greater in magnitude when compared to the movements in the observed spot yields at longer terms. For example, this observation was evidenced in the months following the first quarter of 2020. This observation can, in part, be explained by the inter and extrapolation methodologies, and therefore an investigation of these is included in the scope of the review to consider the effect of curve sensitivity to market movements.

1.4.1.3   The data set used to construct the PA curve consists of government bonds issued by the South African government. The instruments remain in the data set until they mature. This has the effect that sudden discontinuities may arise in the PA curve, at terms, corresponding to the outstanding maturity of these bonds, and this may result as the trading volumes of these bonds decrease as maturity is approached, and differentials are observed in the yields of these bonds as compared to cash instruments at the short end of the PA curve.

1.5   Structure of the document

1.5.1   The remainder of this document is set out as follows: section 2 describes the current PA methodology, including the current data set; section 3 deals with the principles underlying an optimal data set; section 4 investigates alternative curve construction methodologies, including an outline of their relative merits; and section 5 concludes with recommendations.

2.   Description of the current curve construction methodology

2.1   Introduction

2.1.1   The data set of the PA curve consists of the South African government bonds and deposit rates that are used to obtain a risk-free interest rate term structure. The closing yields for instruments in the data set are obtained from the Bloomberg Terminal1 on the first working day of each month.

2.1.2   In documenting the existing PA methodology, investigations and interviews with key role players, at the time when the first curve was published by the then FSB, were conducted.

2.2   The data set and its management criteria

2.2.1   The data set consists of South African government bonds with durations from 1 to 30 years and currency swaps on the South African rand (ZAR) at shorter durations. These instruments are included in the data set until they mature.

2.2.2   The government bonds included in the data set prior to 1 November 2020 were selected by the then FSB Policy Steer group during the development of the SAM framework. These bonds have then been consistently included in the data set, with each bond being held until maturity.

2.2.3   Bonds that are constituents of the GOVI Index (the JSE's government bond index) were considered by the then FSB Task Group for the nominal bond curve. The GOVI Index contains all bonds issued by the South African government, which fall into the top 10 positions (based on market capitalisation[2]) of the ALBI Composite Index.

---

[1] Bloomberg (www.bloomberg.com) is a software platform that provides real-time financial market data.
[2] Market capitalisation is the value of the debt outstanding on the bond.

2.2.4   Bonds that are constituents of the IGOV Index were considered for the real bond curve, that is, the coupon was linked to consumer price inflation (CPI). The IGOV index is a sub-index of the CILI index that encompasses bonds that are issued by the South African government. The CILI index measures the daily movement of inflation-linked bonds which are dually ranked by average liquidity and average market capitalisation.

2.2.5   This approach represented an effective framework for managing the constituents of government bond curves.

2.2.6   Given that there are no government issued inflation-linked bonds at the one-day and the three-month duration points, the one-year swap rate is used as a proxy at both these durations, to ensure the forward rate curve[3] is monotone increasing.

2.2.7   Prior to November 2020, no new government bonds were added to the data set, and that lead to a thinning of the data points used in the mid-terms (between 10 and 30 years), as the maturing bonds exiting the data set were not replaced by new government bond issues, which increased the curve's sensitivity at these terms. The inclusion criteria of the data set were reviewed during October 2020 and this framework is presented in section 3 below.

2.2.8   The change in the data set resulted in a restatement of the PA curve for the October 2020 month-end, and thereafter the PA curve has been published on the revised data set. All the analyses presented in this document is based on the PA curve as at 31 December 2020, with the historic curves presented in section 2.5.

2.2.9   Table 1 below shows the data set in terms of which the nominal PA curve is constructed (as published monthly).

Table 1: Data set - nominal PA curve

Bloomberg ticker symbol	Instrument description	Maturity date
SADR1T	ZAR overnight currency swap	1-day
SADRC	ZAR 3-month currency swap	3-month
SADRF	ZAR 6-month currency swap	6-month
ZARI9M	USDZAR 9-month forward implied yield	9-month
ZARI12M	USDZAR 12-month forward implied yield	1-year
R208	Government Bond	31-03-2021
R2023	Government Bond	28-02-2023
R186	Government Bond	21-12-2026
R2030	Government Bond	31-01-2030
R213	Government Bond	28-02-2031

---

[3] The forward rate curve is a plot of the future one-month interest rates at different terms, derived from forward rates, based on the government bonds used in the PA curve construction.

Bloomberg ticker symbol	Instrument description	Maturity date
R2032	Government Bond	31-03-2032
R2035	Government Bond	28-02-2035
R209	Government Bond	31-03-2036
R2037	Government Bond	31-01-2037
R2040	Government Bond	31-01-2040
R2044	Government Bond	31-01-2044
R2048	Government Bond	28-02-2048

2.2.10   Table 2 shows the data set in terms of which the real PA curve is constructed (as published monthly).

Table 2: Data set - real PA curve

Bloomberg ticker symbol	Instrument description	Maturity date
SASWRY1	ZAR real rate 1yr inflation SWAP	1-day
SASWRY1	ZAR real rate 1yr inflation SWAP	3-month
R212	Government CPI Bond	31-01-2022
R197	Government CPI Bond	7-12-2023
I2025	Government CPI Bond	31-01-2025
R210	Government CPI Bond	31-03-2028
R202	Government CPI Bond	7-12-2033
I2038	Government CPI Bond	31-01-2038
I2050	Government CPI Bond	31-12-2050

2.2.11   Hereinafter, these instruments will constitute the data set in terms of which alternative curve construction methodologies are reviewed.

2.3   Curve interpolation and bootstrapping

2.3.1   The PA curve is based on a linear interpolation method, which is considered relatively uncomplicated to obtain a risk-free interest rate term structure.

2.3.2   The interpolation method uses the Newton-Raphson[4] algorithm to bootstrap the yield curve. Bootstrapping is an iterative process used to derive a zero-coupon curve[5] from the observed market data set. Bootstrapping will produce a risk-free interest rate term structure that exactly prices back all input constituents. The rounding and pricing rules used by the JSE debt market when valuing the bond constituents are applied during this process.

---

[4] The Newton-Raphson method uses an iterative-based approach which estimates the value of the root of a real valued function.

[5] This is a curve that shows rates of return on zero-coupon bonds at different terms to maturity.

2.4   Curve extrapolation and the UFR

2.4.1   A linear extrapolation method is used from the last liquid point of the bonds included in the data set to the end date of the projection period. A period of 150 years was chosen to allow for life insurance liabilities of longer durations.

2.4.2   UFR is the long-term rate to which the forward rate curve is expected to ultimately converge. After this convergence is reached, the forward curve is expected to be constant at this rate for all further periods. For the nominal PA curve, the UFR is 6%, which represents the upper bound of South Africa's long-term inflation target band. The UFR is reached at the 150-year point on the curve.

2.4.3   The speed of convergence (i.e. the rate at which the curve converges to the UFR from the last liquid point) is gradual. For some curve construction methodologies, a 'speed of convergence' is explicitly specified.

2.4.4   On the real PA curve, the UFR is 1.5%. This value is calculated by halving the difference between the upper and lower bound of the inflation target band. The convergence of the UFR is similar to that of the nominal PA curve.

2.5   Plots of historic PA curves

2.5.1   Plots of historic PA curves show the general feature of the curve as well as the effect of the change in the data set during November 2020.

2.5.2   The plot on Figure 1 below shows the nominal PA curve, for two quarter ends over the last three years, including the June 2020 quarter end.

Figure 1: PA curve – nominal (3-year history)
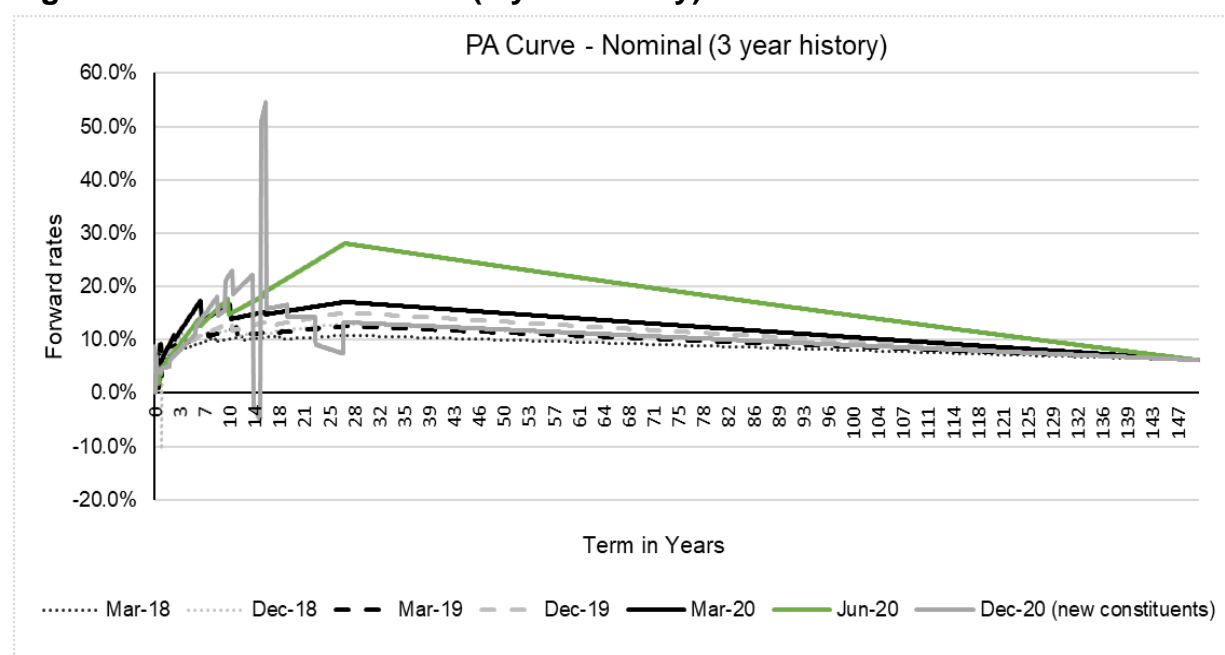
2.5.3   The features of the PA curve before the last liquid point broadly mirror that of a normal yield curve (i.e. not inverted, steep or flat) in that it increases with a high gradient at first, which slows down at terms approaching the last liquid point. The curve, however, slopes downwards after the last liquid point – a result of the UFR being lower than the observable long-term yields (15-to-30-year yields). A linear extrapolation is used in the convergence to the UFR, where the forward curve reaches the UFR at 150 years. The PA curves are not smooth, as they show evidence of discontinuous movements in the liquid part[6] of the curve, due to the interpolation method used in this part of the curve. The forward rates at the 14-to-16-year terms of the curve are negative. This is discussed in paragraph 2.5.5 below. The volatility in the short end of the curve is caused by the swap instruments used, which tend to trade at different yields than those of nearly maturing bonds.

2.5.4   Another notable feature of the PA curves is that it shows a general increase in the level over time – as shown by the level of the 2019 curves (set of thicker dashed curves) being above the level of the 2018 curves (set of thinner dashed curves) in Figure 1 above, and similar for the level of the 2020 curves (set of solid lined curves) being above the level of the 2019 curves. An increase in the level of the PA curves over time may reflect an increase in expectations of future interest rates or inflation rates over time. The noticeable increase in the level of the 2020 curves compared to 2019 curves appear to have been driven by the response of the current methodology and data set to the volatile market conditions in 2020. Following the revised data set implemented during November 2020, the effect on the level of the curve appears to have stabilised at longer durations. However, in the mid-terms, a noticeable shift in the level of the curve remained, which could potentially be linked to the interpolation methodology applied.

2.5.5   Furthermore, it has been observed that the December 2020 curve exhibits some volatility between the 14 and 16-year term (with the negative forward rates applicable for these durations). This is caused by the inclusion of the R209 at the 15-year term, as it has a coupon rate of 6.5% compared to the bonds on either side (i.e. the R2035 and R2037) both having coupon rates of 8.5%. Thus, the trading yields on the R209 tend to be different from the yields on the R2045 and R2037.

2.5.6   The plot in Figure 2 below shows the real PA curve at two quarter ends over the last three years, including the June 2020 quarter end.

---

[6] Terms at which there is observable market bond data.

Figure 2: PA curve – real (3-year history)

PA Curve - Real (3 year history)
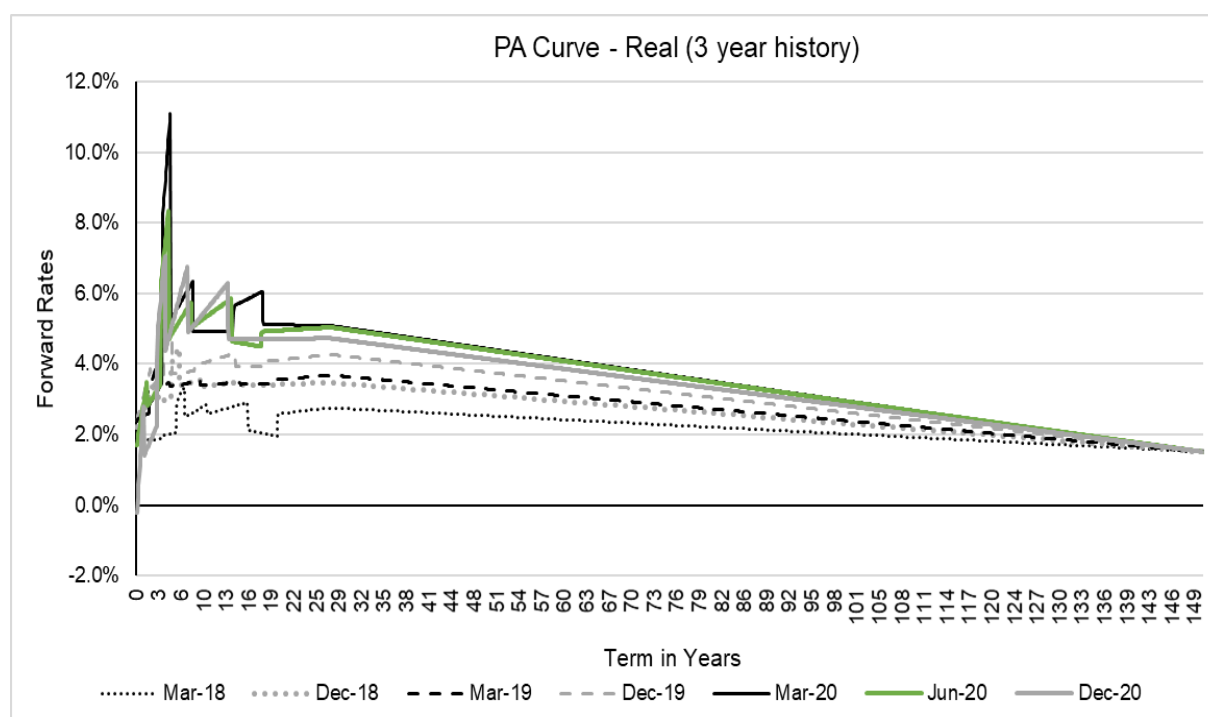
2.5.7 The features of the real PA curve, including the lack of smoothness and the upwards shifts in the level of the curve over time, mirror those of the nominal curve, reflecting stable expectations of future inflation.

2.5.8 The increase in the level of the curves in 2020 is also evident in the real curves, which is likely attributed to the methodology. For the real curve, the data set remained unchanged following the review in October 2020, and it should be considered whether the changes to the data set would be appropriate.

3.   Optimal constituent data set

3.1   Introduction and principles

3.1.1 According to Hagan and West (2008:71), the determination of the number of instruments to include in the data set is not an exact science.

"*Excluding too many, runs the risk of disposing of market information which is actually meaningful, on the other hand, including too many could result in a yield curve which is implausible, with a multitude of turning points, or even a bootstrap algorithm which fails to converge.*"

3.1.1 The then FSB determined that the following principles outline desirable characteristics of a government bond curve to obtain a risk-free interest rate term structure:

- Credit risk: the curve should be credit-risk free;
- Realism: it should be possible for all insurers to earn the specified risk-free rate in a risk-free manner;

- Reliability: the data basis and methods to determine risk-free term structure should be robust, particularly in times of market crisis or turbulence;
- Liquidity: rates should be derived from instruments for which reliable market value is observable from a deep, liquid and transparent market. The term structure should be extrapolated from the longest maturity for which there is sufficient liquidity;
- Objectivity: bid-offer spreads are very low; and
- Simplicity: the derivation of the risk-free rates term structure should be uncomplicated.

3.1.2   These principles are further articulated in Attachment 4 of Prudential Standard FSI 2.2 as a guide to insurers who construct their own curves to obtain a risk-free interest rate term structure.

3.1.3   Based on the principles set out in paragraphs 3.1.1 and 3.1.2 above, a set of constituents is required for which the market value is reliable and derived from a market that is sufficiently deep. In addition, the inclusion of instruments to ensure a plausible yield curve without too many turning points is required. Lastly, a reasonably small set of constituents to ensure successful bootstrapping of the input data set is required.

3.2   Revised 2020 constituent data set

3.2.1   The constituent data set of the nominal PA curve and its management criteria was reviewed during October 2020, to align it with the principles as set out in paragraph 3.1.2 above. The details of the revised criteria are as follows:

3.2.1.1 Inclusion: All vanilla bonds[7] issued by the South African government will be included in the constituent data set when issued or at the month-end following issue if the bond is issued during a month.
3.2.1.2 Management: A quarterly review of the instruments included in the constituent data set will be undertaken to assess the conformity of all bonds with the principles as set in section 3.1, including an assessment of the depth, liquidity and transparency of the market for these instruments.
3.2.1.3 Exclusion: Any bond that exhibits characteristics contrary to the principles set out in section 3.1, such as declines in liquidity or marketability, will be excluded. In addition, bonds will remain in the constituent data set until maturity or until the month-end preceding maturity where these mature during a month.

3.2.2   Table 3 below shows the result of the review of the constituent data set for the nominal PA curve performed during October 2020.

---

[7] Fixed-interest rate bonds.

Table 3: Revised data set – nominal PA curve

Bloomberg ticker symbol	Maturity	Coupon	PA curve – Pre October 2020	PA curve – Post October 2020
R208	31-Mar-21	6.75%	✓	✓
R2023	28-Feb-23	7.75%	✓	✓
R186	21-Dec-26	10.50%	✓	✓
R2030	31-Jan-30	8.00%	✗	✓
R213	28-Feb-31	7.00%	✓	✓
R2032	31-Mar-32	8.25%	✗	✓
R2035	28-Feb-35	8.50%	✗	✓
R209	31-Mar-36	6.50%	✓	✓
R2037	31-Jan-37	8.50%	✗	✓
R2040	31-Jan-40	9.00%	✗	✓
R214	28-Feb-41	6.50%	✗	✗
R2044	31-Jan-44	8.75%	✗	✓
R2048	28-Feb-48	8.75%	✓	✓

3.2.3   A similar review of the constituent data set of the real PA curve was undertaken and it revealed that no changes were required. Table 4 shows the bonds that are included in the data set for the real PA curve.

**Table 4: Current data set – real PA curve**

Bloomberg ticker symbol	Maturity	Coupon
R212	31-Jan-22	2.75%
R197	7-Dec-23	5.50%
I2025	31-Jan-25	2.00%
R210	31-Mar-28	2.60%
R202	7-Dec-33	3.45%
I2038	31-Jan-38	2.25%
I2050	31-Dec-50	2.50%

3.3   Government bonds with undesirable characteristics

3.3.1   From time to time, the government bonds in issue may exhibit characteristics that are undesirable for use by the PA due to these not conforming to the principles set out in paragraphs 3.1.1 and 3.1.2 above. The inclusion criteria of instruments in the data set must be such that any government bonds which display undesirable features, such as low liquidity, can be excluded.

3.3.1   In line with the current criteria set above, the characteristics of the R214 (currently excluded from the data set) and the R209 (currently included in the

data set) were reviewed. With the inclusion of the R214, the PA curve as at 31 December 2020 is as indicated in Figure 3 below.

Figure 3: Nominal PA curve incl. R214
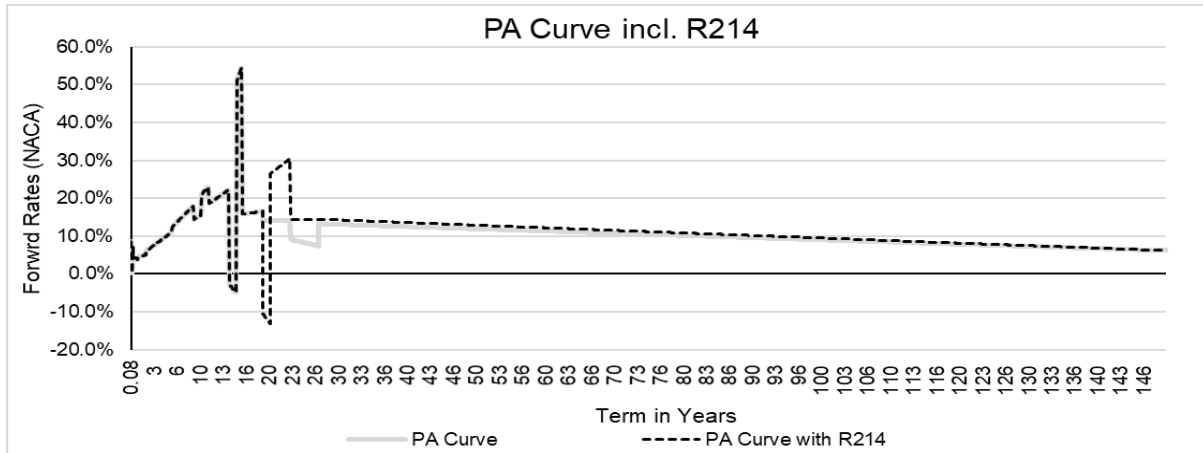
PA Curve incl. R214

3.3.2   The inclusion of the R214 bond in the data set underlying the PA curve introduces volatility in the forward curve around the term which corresponds with the maturity of this bond, which is shown by the additional 'zig-zag' feature in the forward rate curve above, as well as further negative forward rates near the 21-year term. Looking at the coupon rates of this bond shown in Table 3 above, it is noted that that this bond has similar features to the R209 discussed in paragraph 2.5.5. Thus, as the inclusion of the R214 would be contrary to the principles of smoothness and continuity in the forward curve, it should then be excluded from the data set.

3.3.3   Likewise, the exclusion of the R209 is considered. With the exclusion of the R209, the PA curve as at 31 December 2020 is shown in Figure 4 below.

Figure 4: Nominal PA curve excluding R209


PA Curve excl. R209
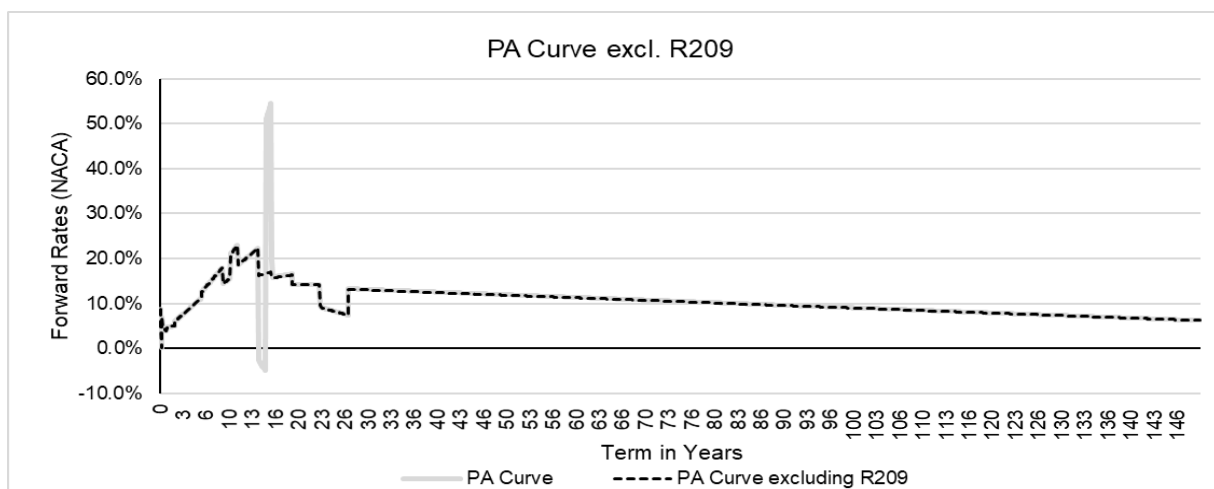
3.3.4   Figure 4 above shows that the exclusion of the R209 removes the 'zig-zag' feature in the current PA curve, addressing the largest discontinuity in the forward rate curve, which enhances the adherence to the principles set out

13

in paragraphs 3.1.1 and 3.1.2 above. Notably, the forward rates remain positive when the R209 bond is excluded. The exclusion of the R209 bond therefore appears plausible in this instance.

3.3.5 Similarly, the inclusion of the three bonds that are in issue but not currently included in the data set of the real PA curve, namely the I2029, I2033 and I2046, was investigated. With the inclusion of these bonds, the real PA curve as at 31 December 2020 is as shown in Figure 5 below.

Figure 5: PA curve – real (CILI15 basis)
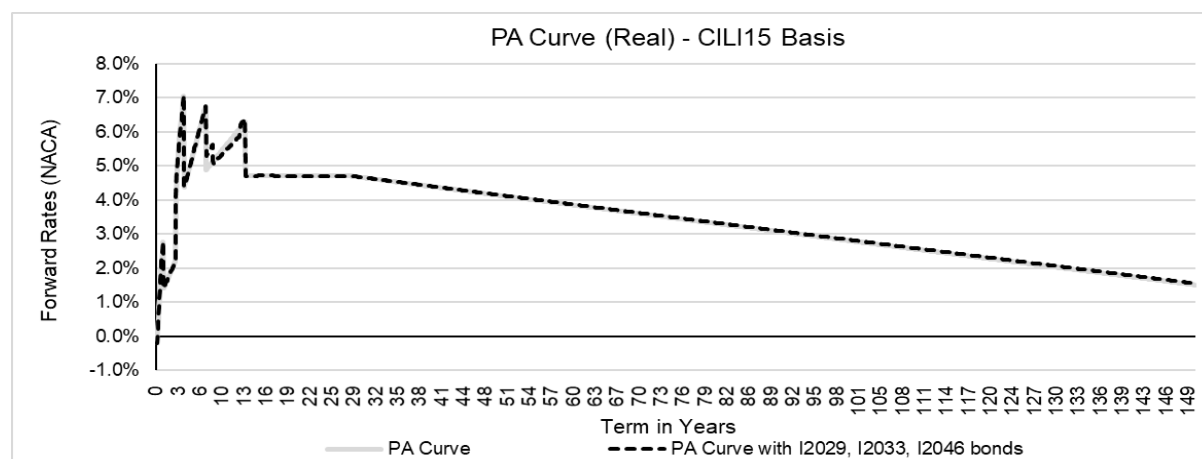
3.3.6 Figure 5 above shows that the inclusion of these bonds has a near negligible effect on the overall level and shape of the real PA curve. Therefore, the inclusion of these bonds is supported.

4. Alternative methodologies

4.1 This section presents alternative curve construction methodologies based on the assumed optimal data set.

4.2 General principles

Barrie and Hibbert (2008:6) suggest the following desirable properties for constructing forward yield curves:

"1. Where a liquid market exists, the yield curve should accurately price that market.
2. The (forward rate) curve should be continuous.
3. The (forward rate) curve should be smooth. That is, its first derivative should be continuous.
4. On average, the variability of long-term (forward) interest rates should be lower than shorter term (forward) rates."

4.2.1   International standards: IAIS ICPs

In ICP 14, published by the IAIS, which deals with valuations, the following paragraphs refer:

"*14.10.1 The solvency regime allows for the time value of money to be recognised in the determination of technical provisions and should establish criteria for the determination of appropriate interest rates to be used in the discounting of technical provisions (discount rates). In developing these criteria, the supervisor should consider the following:*

- *the economics of the insurance obligations in its jurisdiction including their nature, structure and term; and*
- *the extent (if any) to which benefits are dependent on underlying assets.*

14.10.3 To the extent that a risk is provided for elsewhere in the balance sheet by alternative means, there should be no allowance for that risk in the chosen discount rates.

14.10.5 The criteria should also allow appropriate interpolation and extrapolation for non-observable market data and maturities. To provide for consistent, reliable, economic values, the criteria for discount rates should utilise the entire interest rate term structure."

4.2.2   When comparing the alternative curve construction methodologies, the above principles will be considered.

4.2.3   Linear interpolation is used in the conversion of the zero-coupon yields to the instantaneous monthly forward rates. This method has its drawbacks which are documented below.

4.2.4   This review considers the interpolation methods as set out in Hagan and West (2008). Further, it considers an alternative to linear methods, being splines and further builds on this to develop the Monotone convex method. These will be considered in turn as suitable alternatives.

4.2.5   Linear interpolation

4.2.5.1 There are various known linear interpolation methods – a straight-line linear interpolation method on the instantaneous zero rates, which requires knowledge of two points, and the constant rate of change between those two points, as used in the PA curve.

4.2.5.2 The drawbacks of the linear method are widely documented and include discontinuities (or jumps) in the forward rates at terms where the observable instruments have different characteristics.

4.2.5.3 Another drawback is the possibility of negative forward rates. This is an undesirable feature as it implies the possibility of arbitrage opportunities in the interest rate market.

4.2.5.4 The use of splines seeks to address the first of these drawbacks.

4.2.6 Cubic spline – Bessel/Hermite method

4.2.6.1 Splines are a set of polynomials with different coefficients, which are applied in a piecewise manner at different intervals of the curve (Hagan and West, 2008: 74). The polynomials (specified by the coefficients) are arranged such that the resulting curve overlaps the input data set.

4.2.6.2 The Bessel/Hermite method cubic spline specifically ensures continuity by ensuring that the curve is twice differentiable. This means that the overall shape of the curve is also controlled by limiting points of inflection.

4.2.6.3 Figure 6 shows the nominal PA curve (forward) as at 31 December 2020, using the Bessel/Hermite cubic spline method of interpolation. For the purposes of this analysis, the extrapolation method is unchanged (i.e. linear on the instantaneous rates from the last liquid point).

Figure 6: Nominal PA curve – cubic spline
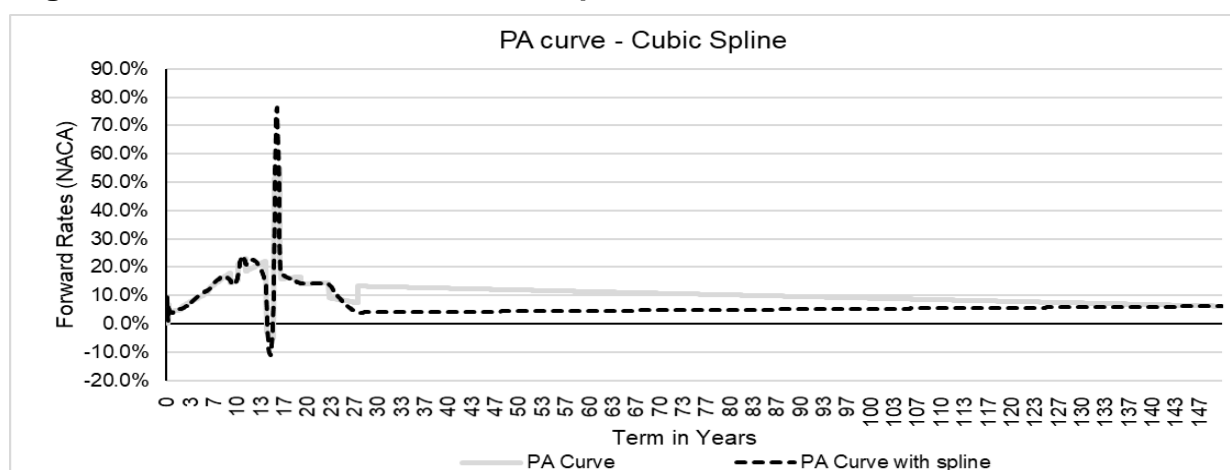
4.2.6.4 The forward rate curve in Figure 6 above shows that although the overall path of the rates mirrors the linear interpolation method, the curve is smoother at the liquid points under the cubic spline.

4.2.6.5 This feature of the cubic spline also ensures smoother transition between the last liquid point and the extrapolation of the curve, as seen at the 27-year term in Figure 6 above.

4.2.6.6 Figure 7 below shows the real PA curve (forward) as at 31 December 2020, using the cubic spline method of interpolation.

Figure 7: Real PA curve – cubic spline

PA curve (Real) - Cubic Spline
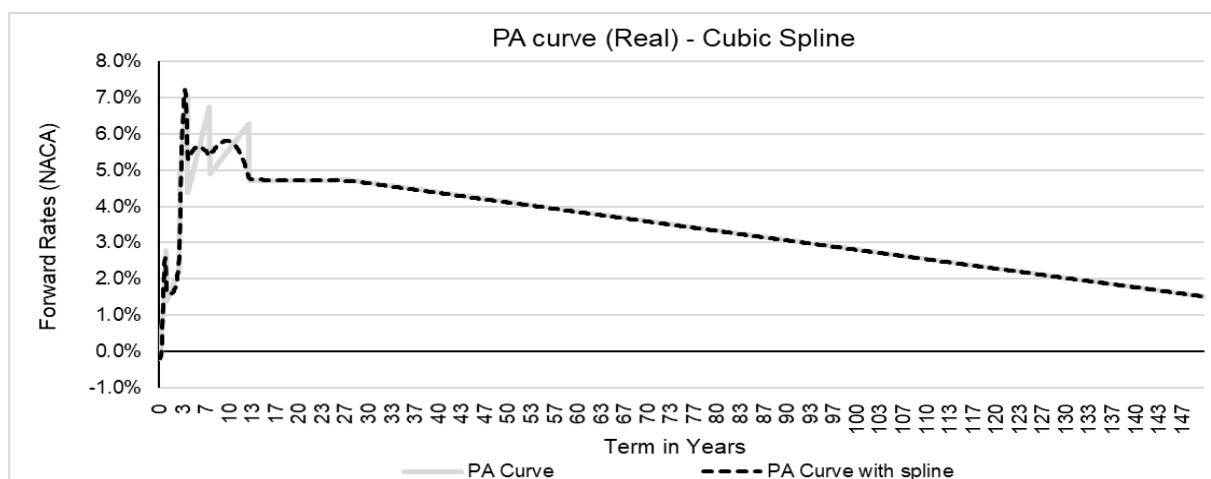
4.2.6.7 The effect of the spline is similar to that shown in the nominal curve since the curve is smoother at the liquid points under the cubic spline. The smooth transition from the last liquid point to the extrapolated portion of the curve is also evident on the real curve.

4.2.6.8 One notable drawback of the cubic spline is that it does not ensure strict positivity in the forward rates. The following method seeks to address this.

4.2.7 Monotone convex method

4.2.7.1 The Monotone convex method imposes a strict positivity of the forwards (except where there are negative yields observed in the market), while ensuring that the original rates are reproduced in the bootstrap.

4.2.7.2 Figure 8 below shows the nominal PA curve (forward) as at 31 December 2020, constructed using the Monotone convex method. For the purposes of this analysis, the extrapolation method is unchanged, that is, linear on the instantaneous rates from the last liquid point.

Figure 8: Nominal PA curve – monotone convex method

PA curve - Monotone Convex Method
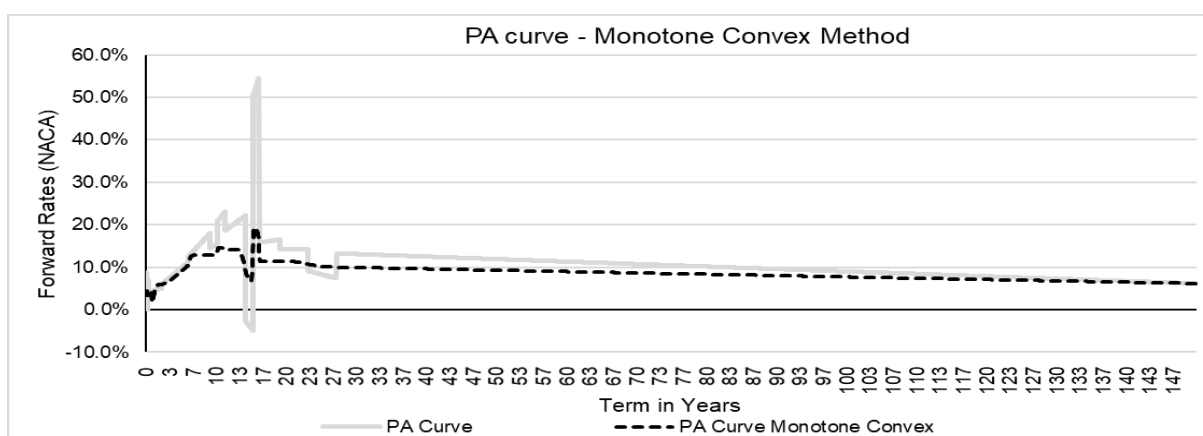
4.2.7.3 Figure 8 above highlights the main strength of this method, as the curve, although maintaining the overall shape of the PA's current method is smooth and consistently above zero. This is a desirable feature in forward rates as

it is consistent with the no-arbitrage feature of a liquid market.

4.2.7.4   It is also worthy to note that the transition between rates at the last liquid point and the point of extrapolation is observed to be smoother, compared to both the linear interpolation method and the cubic spline method.

4.2.7.5   The effect of this method is that the 'hump' between the 7-year and 27-year terms is also muted, similar to the result using the cubic spline. The quicker convergence to the UFR shown in the forward rate curve results in a more gradual decay in the rates under this method.

4.2.7.6   Similarly, the effect of this method on the real curves is assessed. Figure 9 below shows the real PA curve (forward) as at 31 December 2020, constructed using the Monotone convex method. For the purposes of this analysis, the extrapolation method is unchanged, that is, linear on the instantaneous rates from the last liquid point.

Figure 9: Real PA curve – monotone convex method
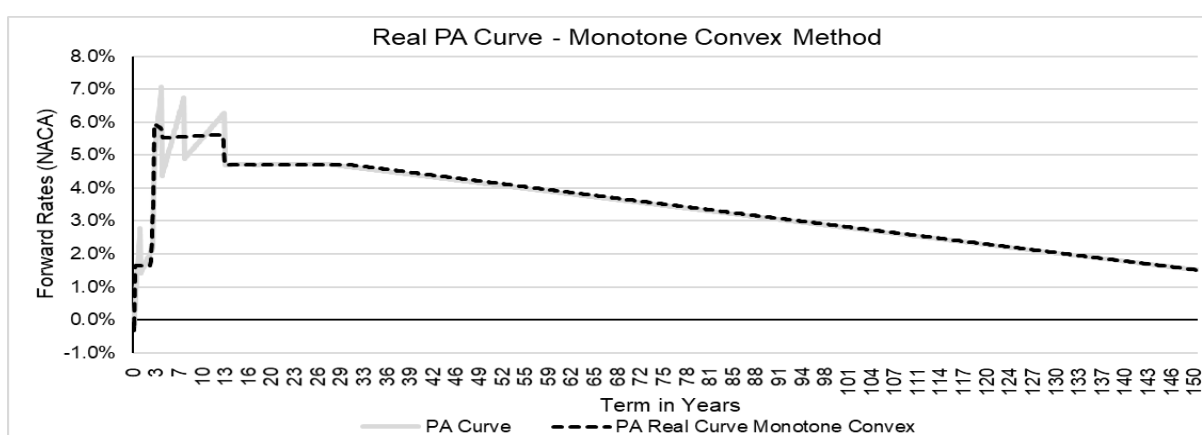
4.2.7.7   Like the nominal curve, the real curve produced by the Monotone convex method exhibits more continuity. It is also notable that although this method ensures positivity in the forward rates, the negative yields observed in the inputs at the short end of the curve are also retained. This exhibits the robust nature (limited sensitivity to small changes in the market) of the method under scenarios where there are negative yields in the market.

4.2.7.8   In conclusion, the Monotone convex method seems to bear the most consistency to the principles as indicated in section 3.1 above, compared to the cubic spline and the linear method. In section 4.4, a set of parametric methods is considered as an alternative approach to the curve construction.

4.3   Curve extrapolation and UFR

4.3.1   Having chosen a suitable set of constituents, a robust interpolation and bootstrapping method, the next feature to consider in the curve construction is the extrapolation, which is itself linked to the decision on the UFR.

4.3.2   In the 2019 study conducted by the Society of Actuaries on extrapolation methods (Akinyemi et al, 2019), the panellists loosely define the UFR as the "*infinite-maturity, unconditional forward rates of interest*".

4.3.3   Barrie and Hibbert (2008: 9) suggest the following principles for estimating the unconditional nominal UFR.

4.3.3.1 The nominal UFR should not be materially affected by short-term economic changes.
4.3.3.2 The UFR should be globally consistent.
4.3.3.3 The approach to estimating the UFR should be simple and easy to understand.

4.3.4   The long-term nominal UFR is typically constructed as the sum of the following components (Akinyemi et al, 2019: 11):

4.3.4.1 the real expected short-term interest rates;

4.3.4.2 the long-term expected inflation;
4.3.4.3 the long-term nominal term premium; and
4.3.4.4 the long-term nominal convexity effect.

4.3.5   The UFR of the current nominal PA curve only considers long-term expected inflation.

4.3.6   Given that the derivation of an UFR should ideally be globally consistent, a real expected short-term interest rate assumption should be added to the long-term expected inflation rate. For this, the European Insurance Occupational and Pensions Authority's (EIOPA) method could be considered, outlined as "*the simple arithmetic mean of annual real rates from 1961 to the year before the recalculation of the UFRs…*"

4.3.7   The resulting UFR can be used in conjunction with the current extrapolation method with a suitable assumption for the period of convergence to the UFR. Alternatively, the Smith-Wilson method can be considered consistently with the EIOPA approach. The Smith-Wilson curve is considered in section 4.4.5 below.

4.3.8   Connected to the choice of the UFR is an assumption on the period to convergence to the UFR. The EIOPA regime sets this as the maximum between the last liquid point plus 40 years and 60 years. For the PA curve, this method would imply a period to convergence of 67 years, as the last liquid point on the current constituents is 27 years (for the R2048) based on the data set as at 31 December 2020. This would mean that after 67 years, the forward curve would be constant or near constant at the asymptote of the UFR, depending on the extrapolation method.

4.4   Parsimonious curve construction methods

4.4.1   In this section, a set of methods commonly used in curve construction is considered – which determine the form of the whole term structure through the setting of a set of parameters.

4.4.2   As a result of their parametric form, these methods are generally considered to be consistent with the principles that have already been outlined and set out in Hagan-West (2008). The Nelson-Siegel, Nelson-Siegel Svensson and Smith-Wilson approaches are considered.

4.4.3   Nelson-Siegel

4.4.3.1   The Nelson-Siegel curve is one of the most popular term structures and it falls within a group of parametric yield curve methods. Nelson-Siegel method is extensively used by central banks and other market specialists since the method is generally efficient in capturing the general shape of the yield curves with the estimation of four parameters.

4.4.3.2   The Nelson-Siegel method is fitted to observations by using the specified functions defined by Nelson and Siegel. The forward and spot curves are a linear combination of three component functions with different shapes, a level (flat curve), sloped and a humped curve to arrive at a best fit.

4.4.3.3   The parameters as indicated in Table 5 below were estimated by minimising the least squares error as part of the forward rate method construction.

Table 5: Nelson-Siegel method parameters

	Nelson-Siegel	
	Nominal curve	**Real curve**
β_0	0.1427	0.0478
β_1	-0.0933	-0.0493
β_2	-0.1393	-0.0285
λ	3.9944	2.7625

4.4.3.4   Figure 10 below shows the nominal PA curve (forward) under the Nelson-Siegel method as at 31 December 2020.

Figure 10: Nominal PA curve – Nelson-Siegel method
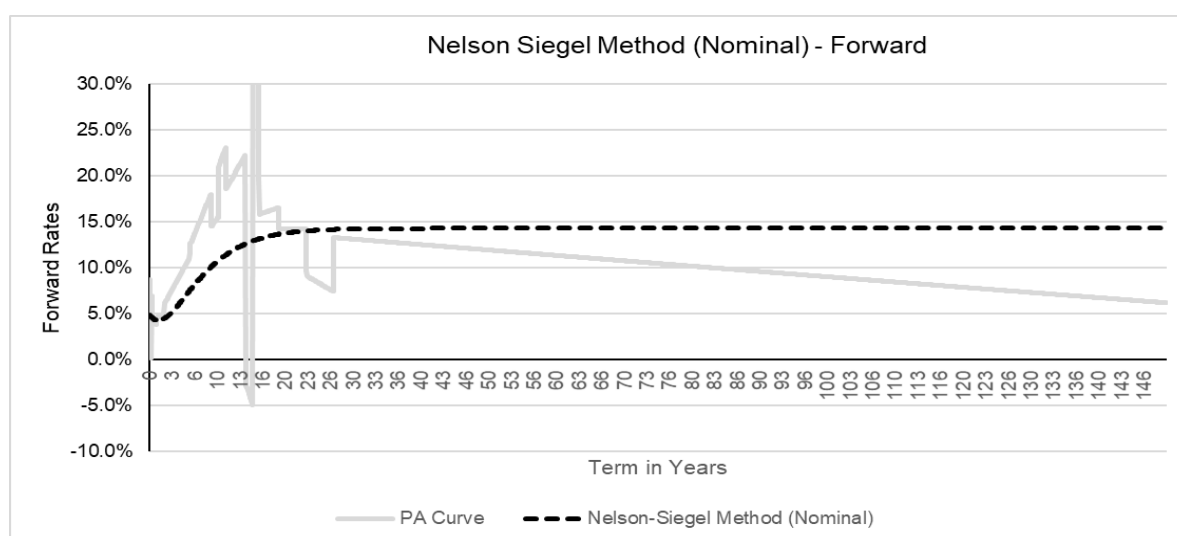
4.4.3.5　　The Nelson-Siegel regression method produces a forward curve that has a more linear slope in the shorter terms and has a more gradual hump shape. The curve tends towards the estimated β_0 parameter (the infinite forward rate) of 14.27% and reaches this point at around the 40-year term. It is noted that the β_0 parameter was unconstrained at the upper bound in the estimation process (i.e., only strict positivity was applied).

4.4.3.6　　Figure 11 below shows the real PA curve (forward) under the Nelson-Siegel method as at 31 December 2020.

Figure 11: Real PA curve – Nelson-Siegel method
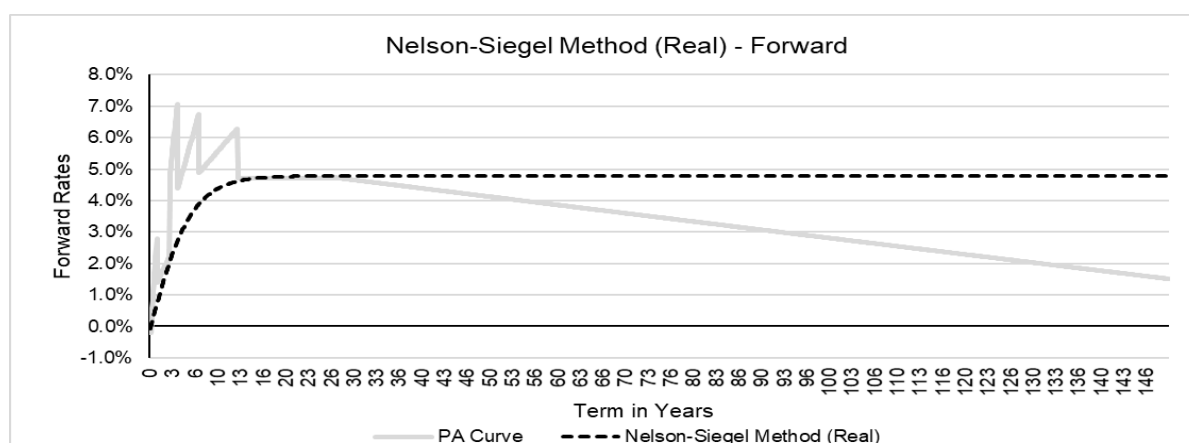
4.4.3.7　　Similarly, the real curve constructed under the Nelson-Siegel method is smoother compared to the PA curve in the terms before the last liquid point. The curve converges to the modelled β_0 parameter of 4.78% at around the 30-year term. As in the nominal case, this parameter was unconstrained at the upper bound in the estimation process.

4.4.4　　Nelson-Siegel Svensson

4.4.4.1　　The Nelson-Siegel Svensson method is an extension of the Nelson-Siegel method, which contains a second hump and allows for an even broader (diverse) set of yield curves to be modelled, and more complicated range of term structure shapes. Similar to the Nelson-Siegel method, Nelson-Siegel Svensson is also extensively used by central banks.

4.4.4.2   The parameters shown in Table 6 below were estimated by minimising the least squares error as part of the model construction (the treatment of the β_0 parameter is as outlined in the Nelson-Siegel method in section 4.4.3).

Table 6: Nelson-Siegel Svensson method parameters

| Nelson-Siegel Svensson		
	Nominal curve	**Real curve**
β_0	0.1412	0.0475
β_1	-0.1049	-0.0737
β_2	-0.1621	-0.0281
β_3	0.0504	0.0667
λ_1	3.5697	2.3486
λ_2	1.68	0.5294

4.4.4.3   Figure 12 below shows the nominal forward curve under theNelson-Siegel Svensson method as at 31 December 2020.

Figure 12: Nominal PA curve – Nelson-Siegel Svensson method
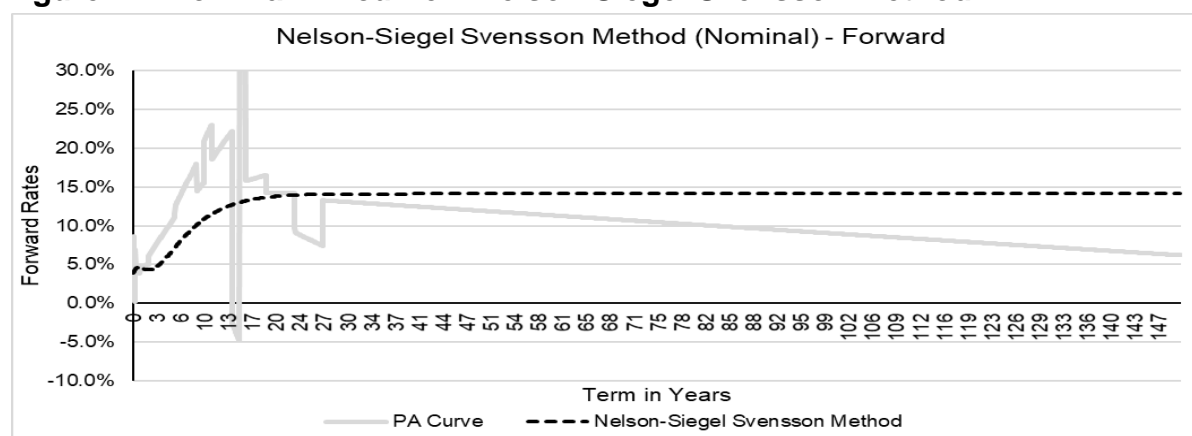
4.4.4.4   The shape of the interest rate term structure under the Nelson-Siegel Svensson – an extension of the Nelson-Siegel method – is not materially different from that determined by the normal Nelson-Siegel method. This is also shown by the similarity in the estimated values of the model parameters. This implies that the extra complexity accommodated for by the Nelson-Siegel Svensson function form may not be needed in fitting the current set of constituents.

4.4.4.5   Figure 13 below shows the real forward curve under the Svensson method as at 31 December 2020.

Figure 13: Real PA curve – Nelson-Siegel Svensson method
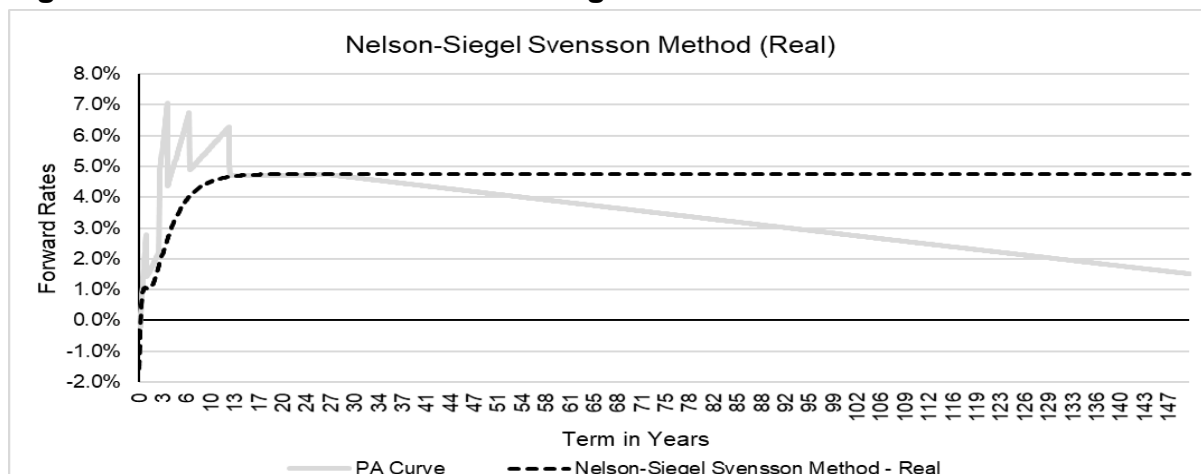
4.4.4.6   As in the nominal case, the Nelson-Siegel Svensson extension outputs a real forward rate curve that is similar to the normal Nelson-Siegel method. This is shown by the estimated parameters in tables 5 and 6 as well.

4.4.5   Smith-Wilson method

4.4.5.1   The Smith-Wilson method is another curve-fitting method that can be used for the interpolation and extrapolation process of the term structure of interest rates. The main advantage of this method is its ability to account for long-term rates through explicit parameters such as the UFR.

4.4.5.2   The above-mentioned ability makes this method to be broadly in-line with the PA's current practice of letting the forward curve converge to a known fixed point, although it has the added sophistication of being able to account for the speed of this convergence. This method was recommended by EIOPA and is currently used in the Solvency II framework.

4.4.5.3   The EIOPA's risk-free rate term structure uses instruments with a maturity from one year onwards. The reason for this is that instruments below 1-year rates may add unnecessary complexity to the calculations. According to EIOPA, this would have an insignificant impact on the rates extrapolated with the Smith-Wilson method, and thus very little impact on the amount of long-term technical provisions.

4.4.5.4 The parameters, as indicated in Table 7 below, were chosen for the construction of the curves.

Table 7: Smith-Wilson method parameters

Smith-Wilson		
	Nominal curve	**Real curve**
Constituents	South African nominal government bonds	South African inflation-linked government bonds
Years to convergence	67 years	77 years
UFR – nominal bond curve	6.00% (NACC)	1.50% (NACC)
Alpha	0.1	0.1
Bootstrapping	Not applicable	Not applicable
Calculation date	31 December 2020	31 December 2020
Volatility adjustment	0%	0%

4.4.5.5 Alpha is a parameter used to adjust the speed of convergence. The extrapolated forward rates converge faster to the UFR for a higher alpha.
4.4.5.6 Figure 14 below shows the nominal bond curve constructed using the Smith-Wilson method (ranging from 1 to 120 years).

Figure 14: Nominal PA curve – Smith-Wilson method
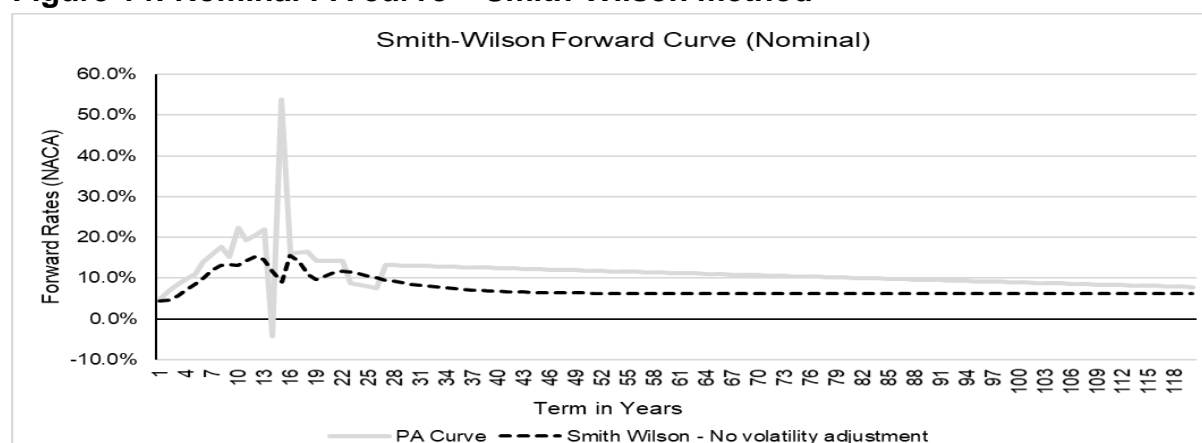
4.4.5.7 The nominal curve, constructed using the Smith-Wilson method, does not spike as sharply in the liquid terms, with the transition from the liquid to the extrapolated portion of the curve also being smoother under this method. The curve then slopes downwards faster towards the UFR, as shown in Figure 14 above, by the steeper downward slope after the last liquid point. The main effect of this is achieving lower rates earlier in the curve compared to a slower progression to the UFR showed by the PA curve.
4.4.5.8 Figure 15 below shows the real spot rates under the Smith-Wilson method (ranging from 1 to 120 years).

Figure 15: Real PA Curve – Smith-Wilson method

Smith-Wilson Forward Curve (Real)
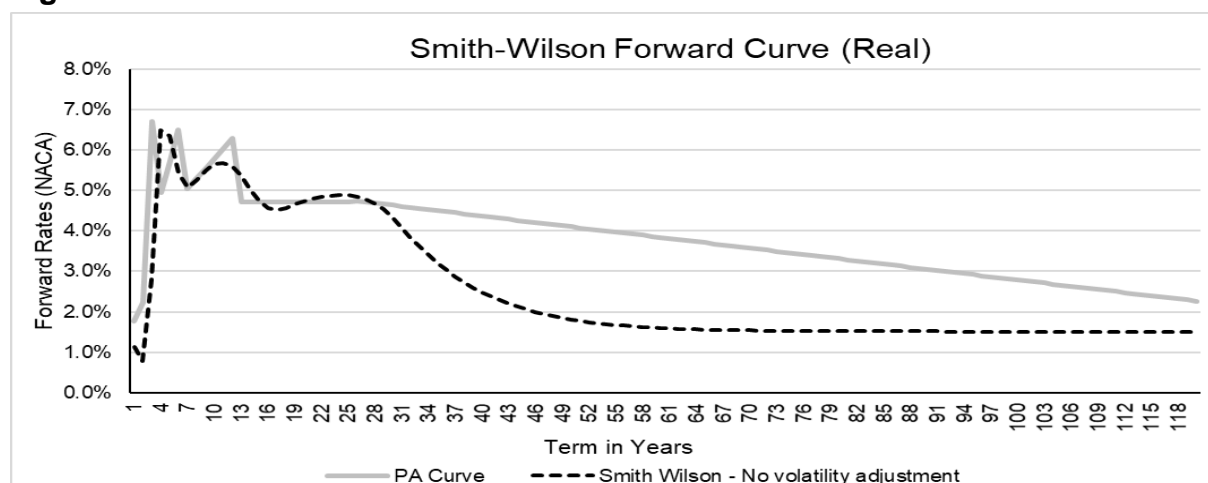
4.4.5.9   Similar to the effects seen above, for the real curve, the main effect is in the extrapolated part of the curve. It contains a steeper downwards slope under the Smith-Wilson method as a result of the earlier convergence to the UFR compared to the current PA method.

4.4.5.10   Finally, Figure 16 below shows a spot rate comparison of the nominal yields under the alternative methods (note that, similar to Figures 14 and 15 above, the Smith-Wilson curve is constructed to an end term of 120 years).

Figure 16: Nominal PA curve comparison

Curve Comparison (Nominal)
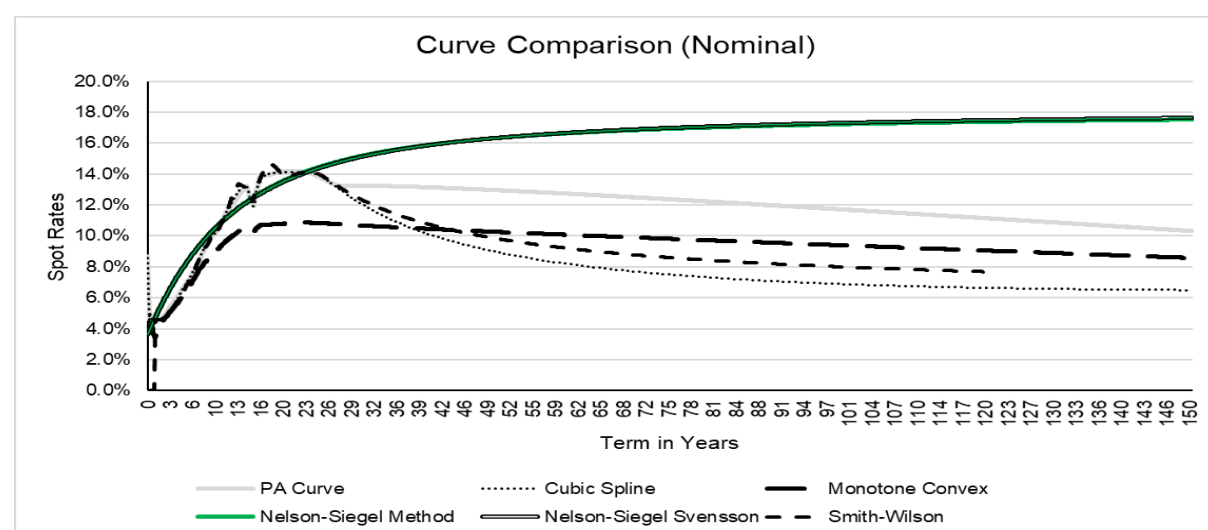
4.4.5.11   This comparison shows that the parametric methods produce the smoothest shape in the liquid part of the curve, with the Monotone convex and Smith-Wilson methods retaining the overall shape of the current   PA   curve – given that these methods do not specify a shape for the curve like the Nelson-Siegel methods.

4.4.5.12   The parametric methods, however, produce higher rates as the estimated forward rates do not converge to an explicit predefined UFR.

4.4.5.13   In the extrapolated portion of the curve, the Monotone convex, Cubic spline and Smith-Wilson methods also decay faster toward the long-term rates compared to the Nelson-Siegel methods. For the Cubic spline, this decay

may not be desirable as it may be a product of negative forward rates.

4.4.5.14 Figure 17 below shows the comparison of the real yields under the alternative methods.

Figure 17: Real PA curve comparison
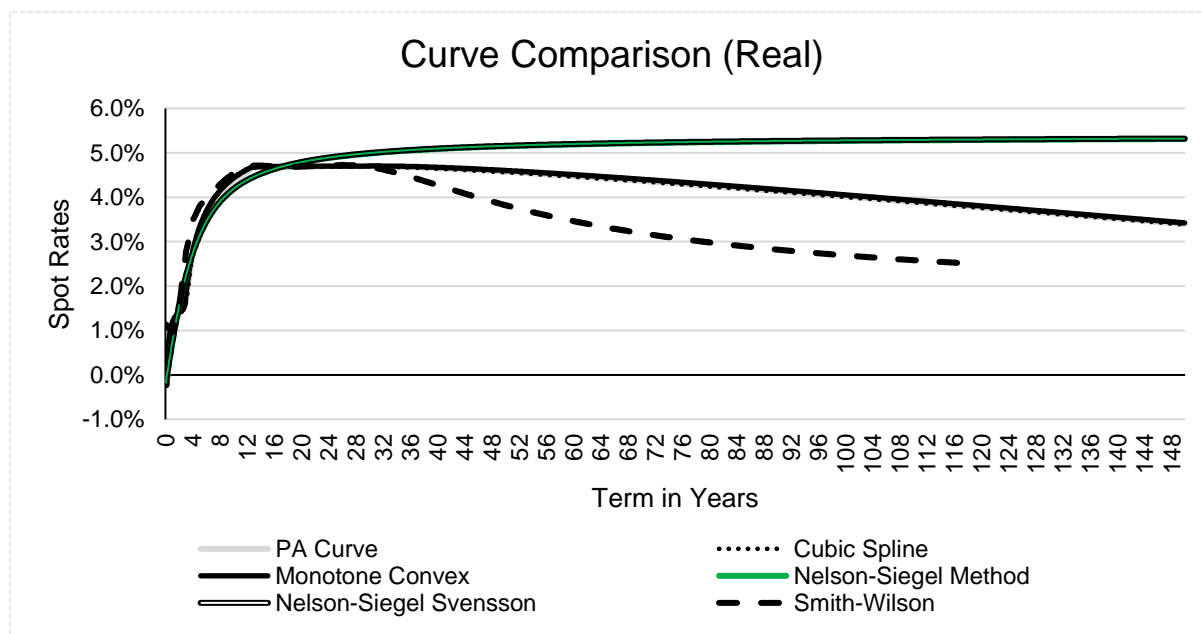
4.4.5.15 The distinctions in the alternative methods for the real curves are less pronounced than those of the nominal curves – with the Nelson-Siegel methods specifying a shape that increases slightly more gradually in the liquid points of the curve and tending towards a higher infinite spot rate.

4.4.5.16 In the extrapolated portion of the curve, the Smith-Wilson method decays much faster than the other methods as a result of the alpha parameter specified in this method.

4.4.5.17 In conclusion, the relative merits of each of the above parametric methods are considered in the section 4.5 below. For the purposes of the PA curve construction and based on the comparisons shown in Figure 16 and Figure 17 above, the Smith-Wilson and Monotone convex methods have the relative advantage of accommodating a quicker convergence to the UFR, which is desirable at the longer terms. On the other hand, the smoother and more gradual increase in the shorter terms of the curve offered by the parametric method forms may also be desirable, especially for discounting shorter term liabilities.

4.5 Advantages and disadvantages of the various methods

4.5.1 The Nelson-Siegel and the Nelson-Siegel Svensson methods are quite different compared to the Smith-Wilson method and the current PA approach in the way they are formulated.

4.5.2   In terms of extrapolation, the Smith-Wilson method and the current PA approach both require the last liquid point and the UFR to be specified, while the Nelson-Siegel method and Nelson-Siegel Svensson use all the observed data inputs to fit the curve and then use the component coefficients to extrapolate the curve beyond the last liquid point.

4.5.3   Table 8 below provides an overview of the advantages of the five methods.

Table 8: Advantages of the five construction methods reviewed

	Advantages
Current PA approach	• Simple method and easy to implement practically and integrate into regular corporate processes • Can be built in-house, eliminating the reliance on vendors
Monotone convex	• Allows for more shapes at various segments of the curve • Ensures positive and smooth forward rates
Nelson-Siegel	• Widely used by central banks and practitioners • Allows the user to match the slope of the fitted curve at the start of the extrapolation • The UFR does not need to be explicitly specified
Nelson-Siegel Svensson	• Widely used by central banks and practitioners • Fits the data slightly better than the Nelson-Siegel method and provides a more accurate yield curve • The UFR does not need to be explicitly specified
Smith-Wilson	• Transparent, and the computing tool is easily accessible • Provides a perfect fit to liquid market data (no smoothing is performed) • Uses the UFR which is reached asymptotically

4.5.4   Table 9 below provides an overview of the disadvantages of the five methods.

Table 9: Disadvantages of the five construction methods reviewed

	Disadvantages	
Current PA approach	• The method can show negative forward rates beyond the liquid part of the curve • The curve can contain discontinuities where the yield is not consistent within an interval	
Monotone convex	• Requires complex estimation of discrete functions at various stages of the calculation	
Nelson-Siegel and Nelson-Siegel Svensson	• Requires the estimation of a set of parameters which may not be straightforward • If accurate estimations are required or when dealing with a complex yield curve, then it might not be the best choice of method to use • Time series of the estimated parameters can be unstable • Assumes forward rates are always positive and the discount factor approaches zero as maturity increases • The over-parameterisation of the method can cause convergence problems	
Smith-Wilson	• The method can show negative forward rates beyond the liquid part of the curve • Expert judgment is needed for the choice of alpha	

5. Recommendations

Based on the review conducted, the findings and recommendations are as follows:

5.1 The constituent data set management proposal, as amended, should be considered for implementation.

5.2 A revision of the nominal UFR could be considered, to include a real interest rate premium to the long-term inflation rate assumption and define a term to convergence, depending on the extrapolation method.

5.3 Notwithstanding the limitations of the current curve construction methodology attributable to the interpolation and extrapolation methods, the current methodology has the advantage of simplicity and easier implementation by industry participants. Therefore, this method should be retained, should the recommendations in paragraph 5.4 not be supported.

5.4 The use of the Monotone convex, Smith-Wilson and Nelson-Siegel Svensson methods as alternative curve construction methodologies should be considered. A quantitative impact study (QIS) should be conducted among industry participants to assess the impact of any proposed alternative methodology prior to concluding on a change to the curve construction methodology of the PA curve.

6. References

- A technical Note on the Smith-Wilson Method:
  http://janroman.dhis.org/finance/Smith%20Wilson/A_Technical_Note_on_the_Smith-Wilson_Method_100701.pdf
- Akinyemi et al, (2019), Society of Actuaries: Yield Curve Extrapolation Methods: https://www.soa.org/globalassets/assets/Files/resources/research-report/2019/yield-curve-report.pdf
- Barrie, A., and J. Hibbert. (2008). A framework for estimating and extrapolating the term structure of interest rates. Exposure draft: http://janroman.dhis.org/finance/Curves%20%20Interpolation/A_Framework_for_Estimating_and_Extrapolating_the_Term_Structure.pdf
- Bond Pricing Formula Specifications, JSE: https://www.jse.co.za/instrument-pricing-specifications
- IAIS ICP14:
  https://iaisweb.org
- Nelson Siegel 1987:
  https://cepr.org/sites/default/files/events/1854_NS_1987.pdf
- Patrick S, Hagan and Graeme West:
  Methods for constructing a yield curve. Wilmott magazine, p 70-81, May 2008
- P.F. du Preez (2011), An investigation into popular methods for constructing yield curves, MSc dissertation, University of Pretoria, Pretoria, 2011: https://repository.up.ac.za/handle/2263/25882
- Prudential Standard 2.2: Valuation of technical provisions:
  www.resbank.co.za
- Solvency Assessment and Management: Pillar 1 - Sub Committee Technical Provisions Task Group Discussion Document 40 (v 3)
- Technical documentation of the methodology to derive EIOPA's risk-free interest rate term structures: https://www.eiopa.europa.eu/content/eiopa-publishes-calculation-ultimate-forward-rate-2020_en

Annexure A: history of data set

Table 10 below shows the set of constituents that was used from the inception of the nominal government bond curve (shown by the ticks). Bonds which have been removed from the data set are marked with an 'x' symbol and new bonds which have been added are marked with a dash (-) symbol in the period before they were added. The table shows the data set at each quarter-end and at months where reconstitutions were made.

Table 10: History of nominal curve data set

Year	2012	2013						2014						2015				2016				2017					2018					2019					2020				
Month	12	3	6	9	10	11	12	3	6	7	8	9	12	3	6	9	12	3	6	9	12	3	6	8	9	12	3	6	9	10	12	3	6	9	11	12	3	6	9	10	12
cash:BA:1d	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
cash:BA:3m	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
cash:BA:6m	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
cash:BA:9m	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
cash:BA:1y	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R201	✓	✓	✓	✓	✓	✓	✓	✓	✓	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
R157	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
R203	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
R204	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	x	x	x	x	x	x	x	x	x	x	x	x	x
R207	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	x	x	x	x	x	x	x	x
R208	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R2023	-	-	-	-	-	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R186	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R213	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R209	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R214	✓	✓	✓	✓	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
R2048	-	-	-	-	-	-	-	-	-	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R2030	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	✓	✓
R2032	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	✓	✓
R2035	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	✓	✓
R2037	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	✓	✓
R2040	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	✓	✓
R2044	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	✓	✓

Table 11 below shows the set of constituents that was used from the inception of the real government bond curve, shown by ticks (✓). Bonds which have been removed from the data set are marked with an 'x' symbol and new bonds which have been added are marked with a dash (-) symbol in the period before they were added. The table shows the data set at each quarter-end and at months where reconstitutions were made.

Table 11: History of real curve data set

Year	2012	2013					2014				2015					2016					2017					2018					2019					2020					2021	
Month	12	3	6	7	9	12	3	6	9	12	3	6	7	9	12	3	6	7	9	12	1	3	6	9	12	3	6	9	10	12	3	6	9	11	12	3	6	9	10	12	3	6
cash:BA:1d	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
cash:BA:3m	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R211	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×
R212	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R197	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
I2025	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R210	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
I2029																				-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
I2033												-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
R202	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
I2038	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
I2046				-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
I2050	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓